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# Min-energy Broadcast in Mobile *Ad hoc* Networks with Restricted Motion

**J.M. Díaz-Báñez · R. Fabila-Monroy ·
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**Abstract** This paper concerns about energy-efficient broadcasts in mobile *ad hoc* networks, yet in a model where each station moves on the plane with uniform rectilinear motion. Such restriction is imposed to discern which issues arise from the introduction of movement in the wireless *ad hoc* networks.

Given a transmission range assignment for a set of $n$ stations $S$, we provide an polynomial $O(n^2)$-time algorithm to decide whether a broadcast operation from a source station could be performed or not. Additionally, we study the problem of computing a transmission range assignment for $S$ that minimizes the energy required in a broadcast operation. An $O(n^3 \log n)$-time algorithm for this problem is presented, under the assumption that all stations have equally sized transmission ranges. However, we prove that the general version of such problem is NP-hard and not approximable within a $(1-o(1))\ln n$ factor (unless NP $\subset$ DTIME$(n^{O(\log\log n)})$). We then propose a polynomial time approximation algorithm for a restricted version of that problem.

**Keywords** Mobile *ad hoc* networks · Restricted motion · Power assignment · Optimization algorithms

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## 1 Introduction

In its simplest form, a computer network may be modeled by a graph. In this graph, computers are represented by nodes, and two nodes are adjacent if the corresponding computers can communicate. Algorithmic issues for this model are very well understood by now. However, technological advancement has made this model insufficient in many instances. Perhaps the most important change was the arrival of mobile wireless devices. A wireless mobile device is any computer that can communicate using radio signals and is also endowed with the ability to change its location. In its more general form, the movement of the nodes is not know in advance (see Camp et al (2002)). In the literature, the most common of these networks are called *Mobile Ad-hoc Networks* (or MANETs), and they are the mobile version of the *Wireless Ad-hoc Networks*.

In this paper we consider the algorithmic issues of networks that lie between these two models, they are wireless ad-hoc networks but with a very restricted motion which is known in advance. Particularly, we focus on issues related to *broadcast* operations and energy optimization.

We briefly review wireless ad-hoc networks. A wireless ad-hoc network consists of a collection of $n$ radio stations, represented by a set of points on the plane $S = \{s_1, s_2, \ldots, s_n\}$, that exchange messages by wireless connections. Each station has an assigned transmission range, and a station $s_j$ can receive a transmission from another station $s_i$ if and only if $s_j$ is within the transmission range of $s_i$. Typically, the stations in such a network have a limited energy resource (battery for example), and consequently, energy efficiency is an important design consideration for these networks (see Clementi et al (2001); Wang and Li (2006)).

The power (energy) needed to correctly transmit data from a station $s_i$ to a station $s_j$ depends on the term $d(s_i,s_j)^\alpha$, where $d(s_i,s_j)$ is the Euclidean distance between $s_i$ and $s_j$, and $\alpha \geq 1$ is the distance-power gradient. In an ideal environment $\alpha = 2$, but depending on the environment conditions it may be as large as 6 (Pahlavan and Levesque (2005)).

In this context, a *broadcast* is a task initiated by a source station in order to disseminate a message to all stations in the wireless network. Broadcasting is a basic network communication task and for that reason has been widely studied (see for example Williams and Camp (2002), Li (2004), Peleg (2007) and Clementi et al (2001)).

We do a small extension of this model to provide movement to the stations. Now each point in $S$ moves on the plane in a straight line with constant speed (which may differ from that of other stations). Thus, the location of every station $s_i$ can be seen as function of time, where $s_i(t)$ is the position of the station $s_i$ at time $t$. The *transmission range* of $s_i \in S$ is represented by a disk $D_{s_i}$ of radius $r_i \geq 1$ centered at $s_i(t)$, for all $t$; we use $r_i$ greater or equal than 1 only to prevent infinitely small transmission ranges. Similarly, a *transmission range assignment* for $S$ is a function $R : S \to \{r \in \mathbb{R} | r \geq 1\}$, so that $r_i = R(s_i)$ for each $s_i \in S$. Consequently, a station $s_j$ can receive a transmission from $s_i$ at time $t$ if and only if $s_j(t)$ is inside the disk $D_{s_i}$, with $i \neq j$.

In regard to broadcast, we assume the following conditions about message transmission: a message transmission can be completed in an instant of time; and if $s_i$ has a message $M$ at time $t$, then it will transmit $M$ to every station lying inside $D_{s_i}$ at any time $t' \geq t$.

As stated earlier, we study the algorithmic issues arising from the introduction of movement to the wireless *ad hoc* networks, but in terms of minimizing the energy required in a broadcast operation. To properly discern those issues, our model is intentionally restricted to simple movements (uniform rectilinear motion) and instantaneous message transmission. From a practical point of view, a model with fixed trajectories could seem rather unrealistic or naive, however such abstraction could still be applied in for example satellite networks (see Resende and Pardalos (2006)) or in models like the one in Zhao et al (2004).

Let $S$ be the set of mobile stations (points) and let $s \in S$ be a source station that generates a message $M$ at time $t_0$. We focus on the following problems:

(1) *broadcast problem*: Given a transmission range assignment $R$ for $S$, to decide if $s$ could perform a broadcast of $M$, in other words, decide if the rest of the stations in $S$ will eventually receive $M$ (at time $t_0$ or later).
(2) *min-equal-range problem*: Find the minimum value $r \geq 1$ needed to broadcast $M$ from $s$, supposing that the transmission range assignment $R$ is such that $R(s_i) = r$, for all $s_i \in S$, i.e. supposing that all stations in $S$ have the same transmission range radius $r$.
(3) $\alpha$*-minsum problem*: Given the distance-power gradient $\alpha \geq 1$, to find a transmission range assignment $R$, such that $s$ could broadcast $M$ and the value $\sum_{s_i \in S} R(s_i)^\alpha$ is minimized; such value represents the overall energy cost of $R$.
(4) *minsum-binary problem*: Is a restricted case of the $\alpha$-minsum problem, where each station could either have a transmission range of radius 1 or cease to transmit.

The first three problems have been widely studied for the static case. The general approach (see Clementi et al (2001) and Wang and Li (2006)) is to obtain a directed transmission graph from the transmission range assignment $R$, and then look for a specific connectivity property of that graph. However, these strategies cannot be used directly in the mobile case, as the transmission of messages also depends on the time of transmission.

This paper is organized as follows: Section 2 presents an algorithm for solving the broadcast problem. Section 3 proposes an $O(n^3 \log n)$-time algorithm to solve the min-equal-range problem. In Section 4 we prove that the $\alpha$-minsum problem is an NP-hard problem, and cannot be approximated in polynomial time within a $(1 - o(1)) \ln n$ factor, unless NP has slightly superpolynomial time algorithms. In the same section, we propose an approximation to the minsum-binary problem, which maintains the NP-hardness property from the $\alpha$-minsum problem. Finally, we present our conclusions in Section 5.

## 2 Broadcast problem

We first recall the *broadcast problem*: Given a set of mobile stations $S$, a transmission range assignment $R$ for $S$, a source station $s \in S$, and a message $M$ generated in $s$ at time $t_0$, to decide if the rest of the stations in $S$ will eventually receive $M$ (at time $t_0$ or later).

To solve the broadcast problem, we propose a method based on Dijkstra's algorithm, which, as a side result, also computes the first time at which each station receives $M$.

Since each $s_i \in S$ moves in a straight line and with constant speed, then $s_i$ can send a message to $s_j$ only within one time interval, with $i \neq j$. We name this interval as *transmission interval from $s_i$ to $s_j$* and is denoted by $I(s_i, s_j)$. If $s_i$ and $s_j$ does not move in parallel lines ($i \neq j$) then $I(s_i, s_j) = [t_a, t_b]$, where $t_a$ is the first instant of time at which $s_j$ lies in $D_{s_i}$, and $t_b$ is the instant of time at which $s_j$ leaves $D_{s_i}$, see Fig. 1.

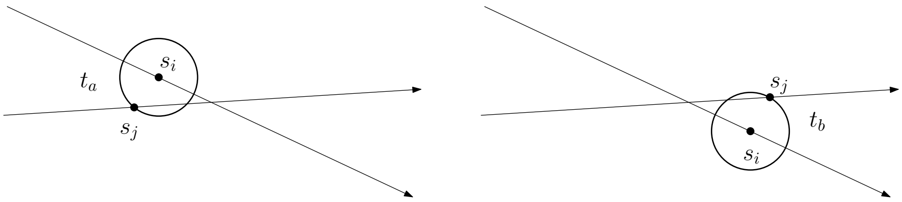


**Fig. 1** Example of times defining $I(s_i, s_j) = [t_a, t_b]$.

Note that, depending on $R$ and the trajectories of $s_i$ and $s_j$, the interval $I(s_i, s_j)$ could be empty; and is always true that $I(s_i, s_j) \subseteq I(s_j, s_i)$ or $I(s_j, s_i) \subseteq I(s_i, s_j)$.

The *connectivity graph* $G_R$, defined by $S$ and $R$, is a directed graph with $S$ as vertex set. An arc from $s_i$ to $s_j$, labeled by $I(s_i, s_j)$, is in $G_R$ if and only if $I(s_i, s_j) \neq \emptyset$. See the left side of Fig. 2 for an example.

If $t_i$ is the first time at which the station $s_i$ receives $M$, then $s_i$ can pass $M$ to another station $s_j$ if and only if $t_i \leq t_b$, where $[t_a, t_b] = I(s_i, s_j)$, that is, before $s_i$ losses (directed) connectivity with $s_j$. This concept can be expressed in $G_R$ in the following way: assign the value $t_i$ to the vertex $s_i$; consider the arc from $s_i$ to $s_j$ (labeled by $[t_a, t_b]$) and assign the time $t_j$ to $s_j$ (the time at which $s_j$ first receives $M$ from $s_i$), where $t_j = t_i$ if $t_i \in [t_a, t_b]$ or $t_j = t_a$ if $t_i \leq t_a$.

Thus, to solve the broadcast problem, it is enough to show that $G_R$ has an induced spanning tree rooted at $s$, with suitable time intervals on the arcs. Therefore, to solve the broadcast problem, we only need to run the following algorithm with $G_R$ and $s$ as inputs:

**Algorithm 1** IS-CONNECTED$(G, s, t_0)$

* $G$ is a connectivity graph and $s$ is the start vertex generating $M$ at time $t_0$. *\

1. Assign the value $t_0$ to the vertex $s$.
2. Assign the value $\infty$ to the other vertices.
3. Run a modified Dijkstra's algorithm from $s$, extracting the vertices from the priority queue according to the time at which they receive $M$.

Dijkstra's algorithm ensures that the vertex coming out of the priority queue has been assigned the minimum distance to the source. In a similar way, IS-CONNECTED assures that each vertex coming out of the queue has been assigned the minimum time at which it receives $M$. Therefore, if the tree obtained from IS-CONNECTED is a spanning tree of $G_R$, then the broadcast from $s$ will succeed (see right side of Fig. 2). In this way, we transform the connectivity problem to a shortest path like problem. As a side result, we also obtain the first time at which each vertex receives the message. The correctness and complexity of IS-CONNECTED follow from those of Dijkstra's algorithm. As $G_R$ could be a complete graph, then the total running time of IS-CONNECTED is $O(n^2)$, and thus we obtain to the following result:

**Theorem 1** *The broacast problem can be solved in $O(n^2)$ time.*

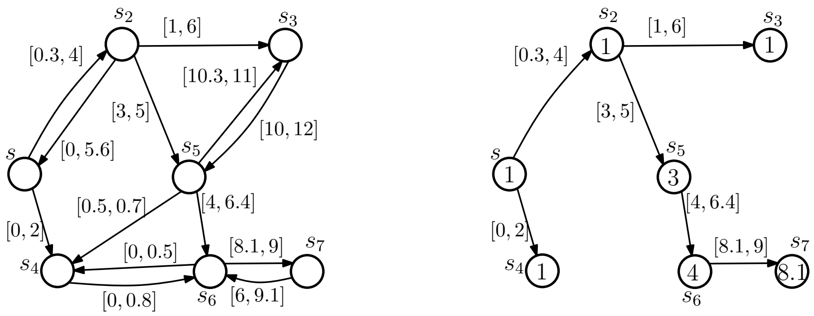


**Fig. 2** The graph $G_R$ for some set of stations $S$ and the spanning tree obtained by the algorithm ($t_0 = 1$).

## 3 Equally sized transmission ranges

In this section we describe an $O(n^3 \log n)$ algorithm to solve the min-equal-range problem. We recall that the problem consists in, given a set of mobile stations $S$, a source station $s \in S$ and a message $M$ generated in $s$ at time $t_0$, to find the minimum value $r \geq 1$ needed to broadcast $M$ from $s$, supposing that the transmission range assignment $R$ is such that $R(s_i) = r$, for all $s_i \in S$.

As the radius of $D_{s_i}$ would be equal to the radius of $D_{s_j}$, we have that $I(s_i, s_j) = I(s_j, s_i)$ and hence we can use them interchangeably. This fact transforms the connectivity graph into an undirected graph.

For any $r \geq 1$, we use the symbol $G_r$ to denote the connectivity graph $G_R$ for the case when $R(s_i) = r$, for each $s_i \in S$. We also denote by $T_r$ to the tree obtained by running the algorithm IS-CONNECTED (from the previous section) on $G_r$ with $s$ and $t_0$. The min-equal-range problem is then reduced to find the minimum radius $r_{MIN}$ for which $T_{r_{MIN}}$ is a spanning tree of $G_{r_{MIN}}$.

The key idea is to calculate a discrete set of possible values for $r_{MIN}$, and then perform a search over this set. We call such set the *critical radii* of $S$, $\mathrm{CR}(S)$, and contains all the radii $r$ where $T_r$ and $T_{r-\varepsilon}$ could differ, for every $\varepsilon > 0$.

Formally, $r$ is a critical radius for $S$ if by taking $R(s_i) = r$ for each $s_i \in S$, one of the following cases arises:

a) Two different stations, $s_i$ and $s_j$, have only one instant $t$ of connection ($I(s_i,s_j) = [t,t]$). See Fig. 3(a) for an example.
b) There is a time $t$ where $I(s_i,s_j) \cap I(s_i,s_k) = [t,t]$ for some stations $s_i$, $s_j$, and $s_k$, with ($s_j \neq s_k$). See Fig. 3(b) for an example.

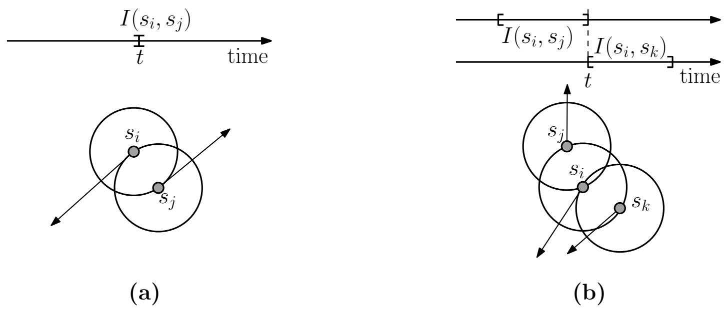


**Fig. 3** Examples of the two cases of critical radii.

A critical radius of type a) corresponds to the insertion of an edge in $G_r$ that was not present in $G_{r-\varepsilon}$, for every $\varepsilon > 0$. A radius of type b) could correspond to the insertion of an edge in $T_r$ that was not present in $T_{r-\varepsilon}$, for every $\varepsilon > 0$; yet the addition of such edge depends on the execution of IS-CONNECTED in $G_r$ and $G_{r-\varepsilon}$.

Given two different stations, $s_i$ and $s_j$, let $d_{i,j}(t)$ be the squared Euclidean distance between $s_i$ and $s_j$ at time $t$. As $s_i$ and $s_j$ move along lines, $d_{i,j}$ is a quadratic polynomial in $t$, and note that $d_{i,j} = d_{j,i}$.

Consider the arrangement of the $n-1$ functions involving $s_i$ ($\{d_{i,j} \mid i \neq j\}$). Any two of these functions intersect at most twice. Therefore, the arrangement contains $O(n^2)$ intersections. Each of the $O(n^2)$ intersections gives us a (squared) radius corresponding to a critical radius of type b), and each of the $n-1$ function minima gives us a (squared) radius corresponding to a critical radius of type a); refer to Fig. 4.

Since we have $n$ different arrangements, then the size of CR($S$) is $O(n^3)$. Assuming that we can obtain the minima of any of these functions, and calculate the intersection of two of these functions in constant time; then we can compute CR($S$) in $O(n^3)$ time.

The algorithm MIN-RADIUS to solve the min-equal-range problem, can be defined as follows.

**Algorithm 2** MIN-RADIUS

1. Compute the set CR($S$).
2. Sort the elements of CR($S$).
3. Look for the minimum radius $r_{MIN}$, such that $T_{r_{MIN}}$ is a spanning tree of $G_{r_{MIN}}$, (use Binary search in CR($S$) and apply the IS-CONNECTED algorithm at each search step).

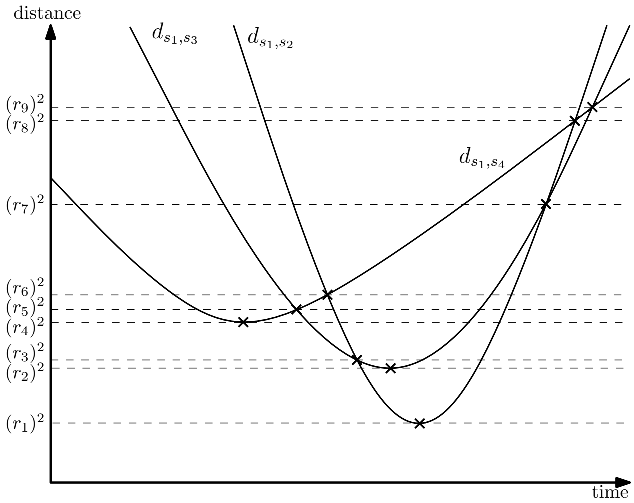


**Fig. 4** Example of the critical radii involving $s_1$.

The time complexity of this algorithm is easy to determine: step 1 takes $O(n^3)$ time, step 2 takes $O(n^3 \log n)$ time, and step 3 takes $O(\log n)$ sub-steps at most, each with a cost of $O(n^2)$ time. Thus, the total time complexity of the algorithm is $O(n^3 \log n)$.

Therefore the next result follows immediately.

**Theorem 2** *The min-equal-range problem can be solved in $O(n^3 \log n)$ time.*

## 4 Optimizing the transmission range assignment

In this section we show that the $\alpha$-minsum is not solvable in polynomial time, unless P = NP , and neither can be approximated in polynomial time within a sublogarithmic factor, unless NP has slightly superpolynomial time algorithms. We prove such statements by reducing the well-known *set cover problem* (Vazirani (2001); Chvatal (1979)) to the $\alpha$-minsum problem (in polynomial time). Additionally, we discuss an approximation algorithm for the minsum-binary problem. Although the proved approximation factor is not satisfactory, it is worth the study for theoretical insights.

### 4.1 The $\alpha$-minsum problem

We first recall that the $\alpha$-minsum problem consists in, given a set of mobile stations $S$, a source station $s \in S$, a message $M$ generated in $s$ at time $t_0$, and the distance-power gradient $\alpha \geq 1$, to find a transmission range assignment $R$, such that $s$ could broadcast $M$ and the value $\sum_{s_i \in S} R(s_i)^\alpha$ is minimized; in other words, to broadcast $M$ minimizing the energy cost.

The static version of the $\alpha$-minsum problem was analyzed in Clementi et al (2001) for several dimensions and different values of $\alpha$. From their results, the NP-hardness of the $\alpha$-minsum is implied for $\alpha > 1$ and stations lying on the plane. However, in our mobile model the $\alpha$-minsum problem is also NP-hard for $\alpha = 1$. In the same way, although the static problem is approximable within a constant factor on the plane (for $\alpha \geq 1$), we will show that the $\alpha$-minsum problem is not approximable within a $(1-o(1))\ln n$ factor, unless NP $\subset$ DTIME$(n^{O(\log\log n)})$, where DTIME$(t)$ is the class of problems for which there is a deterministic algorithm running in time $O(t)$.

An instance of set cover consists of a set $U$, a family $\mathscr{F}$ of subsets of $U$, and a cost function $\mathrm{Cost}: \mathscr{F} \to \mathbb{Q}^+$. The problem is to find a subcollection $\mathscr{F}'$ of $\mathscr{F}$ that *covers* $U$ (every element of $U$ is in at least one element of $\mathscr{F}'$), while minimizing the sum of the costs of its elements, $\sum_{F\in\mathscr{F}'}\mathrm{Cost}(F)$.

It is known that the set cover problem is NP-hard (Vazirani (2001)) and can not be approximated in polynomial time within a $(1-o(1))\ln n$ factor, unless NP $\subset$ DTIME$(n^{O(\log\log n)})$ (Feige (1998)). The same properties can be proven for the $\alpha$-minsum problem, by providing a suitable polynomial time reduction of the set cover problem to the $\alpha$-minsum problem. We now proceed to show such reduction.

### *4.1.1 Reduction*

Let $U = \{u_1,\ldots,u_n\}$, $\mathscr{F} = \{F_1,\ldots,F_k\}$, and $\mathrm{Cost}: \mathscr{F} \to \mathbb{Q}^+$ be an instance of set cover. We assume that every set $F_i$ has a cost greater or equal than 1. If this is not satisfied for this particular instance of set cover, we may add 1 to the cost of each of the sets in $\mathscr{F}$ without changing the optimal solution.

We first describe the initial positions and flight directions of the stations. Afterwards their speeds are specified.

*Initial Positions and Directions.*

Let $N \geq 1$ and a distance parameter

$$P := \Big(N\big(\sum_{F_i\in\mathscr{F}}\mathrm{Cost}(F_i)\big)+n+1\Big)^{1/\alpha}.$$

Let $L_U$, $L_{\mathscr{F}}$ and $L_C$ be three horizontal lines in the plane so that $L_U$ is above $L_{\mathscr{F}}$, and $L_{\mathscr{F}}$ is above $L_C$. We will decide upon the distances between these lines later on. Let $s_{u_1}, s_{u_2},\ldots,s_{u_n}$ be $n$ stations lying in $L_U$ at distance greater than $P$ from each other, and $s_{F_1},\ldots s_{F_k}$ be $n$ stations lying in $L_{\mathscr{F}}$, also at distance greater than $P$ from each other. Both sets of stations $s_{F_i}$ and $s_{u_j}$ will be static.

The stations $s_{u_j}$ represent the elements of $U$, and the stations $s_{F_i}$ represent the elements of $\mathscr{F}$. To represent the fact that a set $F_i$ covers an element $u_j$ we will add a station $s_{c_{i,j}}$ whose original position is in $L_C$. We partition this set of stations into sets of the form $C_j := \{s_{c_{i,j}} | u_j \in F_i\}$. All the stations in $C_j$ will move from their original position towards the station $s_{u_j}$. We choose the original position of each $s_{c_{i,j}}$ so that the line $l_{i,j}$ containing its flight direction, is tangent to the closed disk $D_i$ centered at $s_{F_i}$ with radius $(\mathrm{Cost}(F_i)N)^{1/\alpha}$. Remark that we assume that every station has a minimum transmission range of 1 (that is either it does not transmit or if it does so with a minimum radius of 1). As $\text{Cost}(F_i)$ is greater or equal to 1, so is $(\text{Cost}(F_i)N)^{1/\alpha}$.

Finally, the source $s$ is placed in $L_{\mathscr{F}}$ in any point to the left of all stations $s_{F_i}$ and set its direction along $L_{\mathscr{F}}$ moving to the right. The source will eventually hit every station $s_{F_i}$. Hence, the instance of the $\alpha$-minsum problem will have the set of stations $S = \{s\} \cup \{s_{F_i} | 1 \leq i \leq k\} \cup \{s_{u_j} | 1 \leq j \leq n\} \cup \{s_{c_{i,j}} | 1 \leq i \leq k, 1 \leq j \leq n\}$. See Fig. 5.

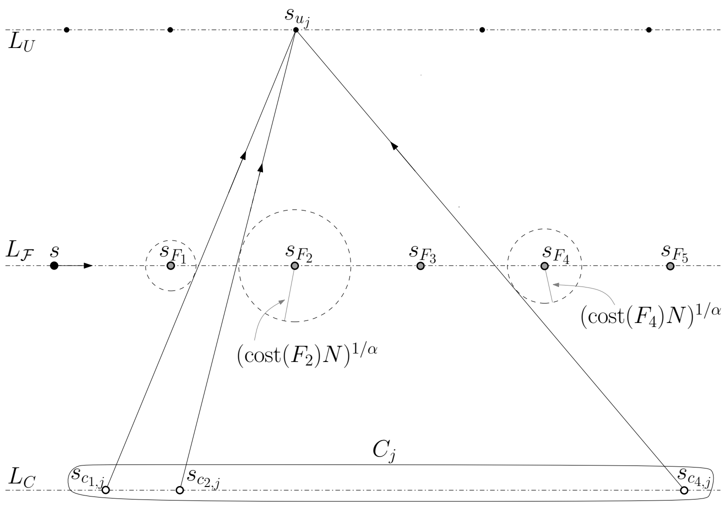


**Fig. 5** Graphical representation of the initial positions and movements (all the involved distances are greater than $P$, except for those explicitly defined).

We now decide upon the separation of the lines $L_U$, $L_{\mathscr{F}}$ and $L_C$. Set them apart from each other so that:

- The lines $L_u$, $L_{\mathscr{F}}$ and $L_{\mathscr{F}}$ are at a distance greater than $P$ from each other.
- No station $s_{c_{i,j}}$ is ever at distance less than $P$ from a station $s_{F_r}$, unless $i = r$.
- No station $s_{c_{i,j}}$ is ever at distance less than $P$ from a station $s_{u_k}$, unless $j = k$.

*Flight Speeds*

Note that the only stations moving are the source and those originally at $L_C$. The flight directions of these stations have been specified above, we need only to decide on their speeds.

To ease the exposition, we partition the set of moving stations into $n+1$ subsets $S_0, S_1, \ldots, S_n$. We refer to these subsets as stages. Stage $S_0$ will consist of solely of the source $s$ passing above each $s_{F_i}$; stage $S_j$ will consist of the set of stations $C_j$ going to the station $s_{u_j}$ for $1 \leq j \leq n$.

The speeds of the stations at each stage are chosen so that:

- For $j > 0$, all stations in stage $S_j$ arrive at station $s_{u_j}$ at the same time.

- The source $s$ is always at a distance greater than $P$ from every station $s_{c_{i,j}}$ and generates the message $M$ (the one to be broadcasted) before passing over any station $s_{F_i}$.
- No station $s_{c_{i,j}}$ in stage $S_j$ will touch the disk $D_i$ before a station $s_{c_{k,l}}$ in a previous stage (i.e. $j > l$) touches the disk $D_k$.
- After a station $s_{c_{i,j}}$ in stage $S_j$ touches the disk $D_i$, no station of a later stage will be at distance less or equal than $P$ from $s_{c_{i,j}}$.

**Observation 1** *The previous construction can be done in polynomial time and we can observe the following:*

- *If a station $s_{c_{i,j}}$ receives the message from a station not in $C_j$ and different from $s_{F_i}$, then the transmission range of such station must be greater than $P$.*
- *If a station $s_{u_j}$ receives the message from a station not in $C_j$, then the transmission range of such station must be greater than $P$.*

*4.1.2 NP-hardness and Inapproximability Results*

The following two lemmas guarantee the NP-hardness of the $\alpha$-minsum problem. Intuitively speaking, a range assignment $R$ that solves the $\alpha$-minsum problem in the previous construction (allowing a broadcast from $s$ and minimizing $\sum_{s_i \in S} R(s_i)^\alpha$), selects a set of stations $S_{\mathscr{F}'} \subseteq \{s_{F_i} | 1 \leq i \leq k\}$ where $R(s_{F_i}) = (\mathrm{Cost}(F_i)N)^{1/\alpha}$ for each $s_{F_i} \in S_{\mathscr{F}'}$. Such set maps to a set $\mathscr{F}' \subseteq \mathscr{F}$ of minimum cost that covers $U$, solving then our instance of the set cover problem.

**Lemma 1** *For every $\alpha \geq 1$ and $N \geq 1$, a function $f$ exists that transforms an instance $x$ of set cover into an instance $f(x)$ of the $\alpha$-minsum problem in polynomial time and satisfying that:*

1. *If there is a solution to $x$ of cost $w$, then there is a solution to $f(x)$ of cost $Nw + (n+1)$.*
2. *If there is a solution to $f(x)$ of cost $w'$ then there is a solution to $x$ of cost at most $(w' - (n+1))/N$.*

*Proof* Let $U = \{u_1, \ldots, u_n\}$, $\mathscr{F} = \{F_1, \ldots, F_k\}$, $\mathrm{Cost} : \mathscr{F} \to \mathbb{Q}^+$ be an instance of set cover. The function $f$ will be the above construction, obtaining then an instance of the $\alpha$-minsum problem.

First assume that there is a solution to this instance of set cover of cost at most $w$. Thus there is a subset $\mathscr{F}' = \{F_{1'}, \ldots, F_{k'}\}$ of $\mathscr{F}$ that covers $U$, such that

$$\sum_{F_{i'} \in \mathscr{F}'} \mathrm{Cost}(F_{i'}) = w.$$

Note that the source $s$ may pass the message $M$ to all stations $s_{F_i}$ in $L_{\mathscr{F}}$ with a transmission range radius of 1. Set the transmission range of every station $s_{F_{i'}}$ to $(\mathrm{Cost}(F_{i'})N)^{1/\alpha}$ and turn off the transmission range of all other stations $s_{F_i}$. Each station $s_{F_{i'}}$ transmits $M$ to every station $s_{c_{i',j}}$ with a cost of

$$((\mathrm{Cost}(F_{i'})N)^{1/\alpha})^\alpha = \mathrm{Cost}(F_{i'})N;$$

hence the total cost for this message transmission is $\sum_{F_{i'} \in \mathscr{F}'}(\mathrm{Cost}(F_{i'})N) = Nw$. Since for every element $u_j$ there exists a set $F_{i'}$ that covers $u_j$, the station $s_{c_{i',j}}$ received the $M$ from $s_{F_i}$ and may transmit it to the station $s_{u_j}$ and all other stations $s_{c_{k,j}}$ arriving at $s_{u_j}$. Choose one of these stations per element $u_j$ and set its transmission range to 1. We have the following conditions:

1. All stations $s_{F_i}$ received $M$ from the source $s$;
2. all stations $s_{c_{i,j}}$ also received $M$, either from $s_{F_i}$ or from a station in $C_j$;
3. all stations $s_{u_j}$ receive $M$ from some element in $C_j$.

Therefore, the constructed range assignment is a solution for this instance of the $\alpha$-minsum problem; its cost is $Nw + (n+1)$.

Now assume that there is a solution to this instance of the $\alpha$-minsum problem of cost $w'$. By Observation 1, if a station $s_{c_{i,j}}$ receives $M$ from a station other than $s_{F_i}$ or a station not in $C_j$ then the transmission range of that station must be greater than $P = (N(\sum_{F_i \in \mathscr{F}} \mathrm{Cost}(F_i)) + n + 1)^{1/\alpha}$ and its associated cost greater than $N(\sum_{F_i \in \mathscr{F}} \mathrm{Cost}(F_i)) + n + 1$. In this case

$$w' > N(\sum_{F_i \in \mathscr{F}} \mathrm{Cost}(F_i)) + n + 1$$

and we may take all of $\mathscr{F}$ as a solution to this instance of the set cover problem; its cost would be at most $(w' - (n+1))/N$. Likewise if a station $s_{u_j}$ receives $M$ from a station not in $C_j$. Assume then that all stations $s_{c_{i,j}}$ received $M$ either from station $s_{F_i}$ or from a station in $C_j$, and that all stations $s_{u_j}$ received $M$ from a station in $C_j$ . Note that the source must have a transmission range of at least 1. Also for every station $s_{u_j}$ at least one station of $C_j$ had transmission range of at least 1. This accounts for at least $(n+1)$ of the cost. The remaining cost is of at most $w' - (n+1)$. Therefore the total cost of stations $s_{F_i}$ whose transmission range is greater than zero is at most $w' - (n+1)$. So if we choose $\mathscr{F}'$ to be the subset of $\mathscr{F}$ of all $F_i$ such that $s_{F_i}$ has a transmission range greater than 0 we obtain a set that covers $U$ whose cost is at most $(w' - (n+1))/N$. □

**Lemma 2** *For every $\alpha \geq 1$ and $N \geq 1$, a function $f$ exists that transforms an instance $x$ of the set cover problem to an instance $f(x)$ of the $\alpha$-minsum problem in polynomial time and satisfying that the optimum value $\mathrm{OPT}(f(x))$ of a solution to $f(x)$ is equal to $\mathrm{OPT}(x)N + (n+1)$, where $\mathrm{OPT}(x)$ is the optimal value of a solution to $x$.*

*Proof* Let $U = \{u_1, \ldots, u_n\}$, $\mathscr{F} = \{F_1, \ldots, F_k\}$, $\mathrm{Cost} : \mathscr{F} \to \mathbb{Q}^+$ be an instance of set cover. Let $f$ be as in Lemma 1. By Lemma 1, a solution to $f(x)$ exists with cost $\mathrm{OPT}(x)N + (n+1)$. If $f(x)$ has a solution of cost less than $\mathrm{OPT}(x)N + (n+1)$, by the same Lemma, $x$ would have a solution of less than $\mathrm{OPT}(x)$, a contradiction. Therefore $\mathrm{OPT}(f(x))$ is equal to $\mathrm{OPT}(x)N + (n+1)$ as claimed. □

The previous lemmas also help to prove the next result, regarding to approximation algorithms.

**Theorem 3** *Let $\alpha \geq 1$. If there exists a polynomial time $\beta$-approximation algorithm for the $\alpha$-minsum problem, then there exists a $\beta$-approximation algorithm for the set cover problem.*

*Proof* Let $x$ be an instance of the set cover problem. Choose $N$ be large enough so that $(\beta-1)(n+1)/N$ is less than the minimum difference between any solution of $x$. Assume that there exists a polynomial $\beta$-approximation algorithm for the $\alpha$-minsum problem. Let $f$ be as in Lemma 2. If $w'$ is the cost solution produced by the algorithm, then by Lemma 2:

$$w' \leq \beta \mathrm{OPT}(f(x)) = \beta(\mathrm{OPT}(x)N+n+1).$$

By Lemma 1 there is a solution to $x$ of cost at most:

$$(w'-(n+1))/N \leq \beta \mathrm{OPT}(x) + (\beta-1)(n+1)/N.$$

And by choice of $N$ this is at most $\beta \mathrm{OPT}(x)$. □

The set cover problem is not approximable within a $(1-o(1))\ln n$ factor, unless NP $\subset$ DTIME$(n^{O(\log\log n)})$ (Feige (1998)), therefore, as a consequence of Theorem 3, we arrive to the following result:

**Theorem 4** *The $\alpha$-minsum problem is NP-hard and it is not approximable within a $(1-o(1))\ln n$ factor, unless* NP $\subset$ DTIME$(n^{O(\log\log n)})$.

### 4.2 The minsum-binary problem

The problem of determining a good approximation factor to the $\alpha$-minsum problem seems to be non-trivial. We now discuss an approximated solution to the minsum-binary problem that can be computed in polynomial time. Although the achieved approximation factor is high and it does not have a real merit in practice, it has interest from a theoretical point of view.

First we recall that the minsum-binary problem consists in, given a set of mobile stations $S$, a source station $s \in S$, and a message $M$ generated in $s$ at time $t_0$, to find the minimum number of stations that must be turned on (have transmission range radius set to 1), in order to broadcast $M$. We also assume that the rest of stations do not transmit (or by abuse of notation, have a transmission range radius of 0).

In the same way as with Theorem 4, the construction of the previous subsection and Theorem 3 imply the following result:

**Theorem 5** *The minsum-binary problem is NP-Hard and it is not approximable within a $(1-o(1))\ln n$ factor, unless NP $\subset$* DTIME$(n^{O(\log\log n)})$.

Our approximation algorithm is based on the greedy approximation algorithm for the set cover problem (Chvatal (1979)). Broadly speaking, at each step we maximize the ratio of the number of new stations that receive the message over the cost of transmitting the message to these stations. Specifically, we choose the station $s_i$ and the time $t$, that maximizes the ratio of the number of new stations that will get the message from $s_i$ after time $t$ over the minimum number of stations that must be turned on for $s_i$ to receive the message by time $t$. We consider only those times where a station $s_j$ enters in the transmission range of $s_i$.

Formally, let us suppose that $s_i$ is turned on and let $t_{i,1} \le t_{i,2} \le \ldots \le t_{i,k_i}$ be the times at which other stations enter in the transmission range of $s_i$. Let $S_{i,k}$ be the set of stations that would be in the transmission range of $s_i$ at time $t_{i,k}$ or after. We then define the cost of $S_{i,k}$ to be the minimum number of stations that must be turned on in order for $s_i$ to get $M$ by time $t_{i,k}$. We denote it by $\mathrm{Cost}(S_{i,k})$.

This cost function can be computed in polynomial time as follows: Suppose that all the stations are turned on, and let $t_1 \le t_2 \le \ldots \le t_p$ be the times at which communication is gained between any two stations. For any of these times $t_k$, we define $G(t_k)$ as the subgraph of $G_1$ (from Section 3), that has an edge joining vertices $s_i$ and $s_j$, if $s_i$ is ever at a distance at most 1 of $s_j$ after time $t_k$. Also let $T(t_k)$ be the tree obtained by running IS-CONNECTED (from Section 2) with inputs $G(t_k)$, $s$ and $t_0$.

Now observe that, if $s_i$ has the message $M$ at time $t_k$, then $\mathrm{Cost}(S_{i,k}) - 1$ is exactly the length of the path between $s$ and $s_i$ in $T(t_k)$. Therefore, by computing the trees $T(t_1), T(t_2) \ldots, T(t_p)$ we obtain the cost of every set $S_{i,k}$ in polynomial time, and hence the approximation algorithm could be stated as follows:

**Algorithm 3** APPROX-MINSUM-BINARY

1. While $S$ has stations not yet ''covered''
2. Select the $S_{i,k}$ with the maximum
   $\mathrm{ratio}(S_{i,k}) = (\#$ stations in $S_{i,k}$ not yet ''covered''$)/\mathrm{Cost}(S_{i,k})$.
3. Mark the stations in $S_{i,k}$ as ''covered''.
4. Turn on $s_i$.

From the analysis in Chvatal (1979), our greedy algorithm will be at most at a factor of the optimal solution of this set cover instance. However, for the minsum-binary problem, the produced solution will be within a bigger factor from the optimal solution. With an extra analysis it is possible to obtain the approximation factor $(\ln n + 1)\mathrm{OPT}$, where OPT is the value of the optimal solution.

## 5 Conclusions

In this paper we worked on problems related to energy-efficient broadcasts in mobile *ad hoc* networks on the plane. Particularly, we were interested in the algorithmic issues arising from the introduction of movement. To properly discern those issues, we have restricted to uniform rectilinear motion and instantaneous message transmission.

Surprisingly enough, some of the proposed problems could still be solved in polynomial time in such environment, like the broadcast problem and the min-equal-range problem. As expected, some other problems got harder in their mobile version than in their static version. The $\alpha$-minsum problem is NP-hard even when $\alpha = 1$ and its minimum approximation factor grew from constant (in the static case) to logarithmic on the number of stations (see Theorem 4).

We believe that very little can be accomplished by approximation algorithms for the $\alpha$-minsum problem. In fact, we conjecture that cannot be approximable within an even bigger factor than logarithmic. On the other hand, we must note that the IS-CONNECTED algorithm could led to a simple heuristic method for the $\alpha$-minsum problem, by applying a tabu search strategy (Glover and Laguna (1997)).

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